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NONLINEAR OBSERVERS FOR CONTINUOUS FERMENTATION PROCESSES

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Abstract: This paper deals with nonlinear observers for a microbial growth rate model in continuous fermentation processes. The nonlinear model is steered to an "extended" bilinear system by change of state variables and output injection. One particular case is presented for an analytical expression of the specific growth rate. Theoretical stability and convergence properties are also analyzed. The effectiveness of the proposed observers is illustrated on simulated data.

Keywords: Bioprocess, specific growth rate, nonlinear and "extended" bilinear systems, observer, output injection.

1. Introduction

In recent years, a great interest is devoted to the biotechnological processes, especially in pharmaceutical and food industries. In order to design, to test and to define the optimal operating conditions of these processes, the bioengineer has to elaborate a mathematical model accounting for their dynamical behavior. This task is especially difficult due to the bioprocesses complexity (nonlinearity, nonstationarity, ...).

In particular, a parameter such as specific growth rate depends on the temperature, pH, substrate, biomass, inhibitor, ... This parameter is either on-line estimated (considered as an unknown time varying parameter [5]) or given by a mathematical formulation. In the second case the problem is to choose the most suitable expression. This is previously studied in [3].

Another essential difficulty lies in the absence, in most cases, of cheap and reliable instrumentation suited to realtime monitoring. To date, there are few sensors able to provide reliable on-line measurements of the biological variables required to implement automatic control strategies. The main variables (biomass, substrate) generally need determining through laboratory analysis. The cost and duration of the analysis obviously limit the frequency of the measurements [5].

It is therefore worthwhile to consider the problem of reconstructing the state of the system (i.e. observability analysis [10]) from the only on-line available measurements. Contrary to linear systems for which the observability is independant of the applied input (if the system is observable then this is true for all inputs), for a nonlinear system, generally there exist inputs called "bad" inputs, which yield it unobservable [7]. It is interesting for the bioprocess basic model to characterize the bad inputs, especially when we have to estimate the unmeasurable state variables. We have then to construct observers which reasonably operate only with the universal inputs or "good" inputs. Bad inputs are obviously disturbing: they constitute the singularity of the observation problem. We can find in [2] a class of inputs which yield the bioprocess basic model unobservable. Therefore, since observability depends on the specific growth rate expression, when we measure the biomass, if the specific growth rate does not depend of the substrate then the null input (null dilution rate or batch process) yield the bioprocess unobservable.

This paper deals with nonlinear observers for a continuous bioprocess model. We recall in §.2 some important results and give basic definitions. For universal inputs, we propose in §.3 an observer by using an analytical expression for the specific growth rate. A numerical example tested on simulation illustrates in §.4 the observers construction for biotechnological processes.

This study is in the extensive scope researches about computer aided modelling of bioprocesses [6].

2. Preliminaries

We recall briefly here some important results. One can find in [12,9,1] some asymptotic observers for bilinear systems. In [11], the authors discussed the question of observers construction for those nonlinear systems which can be transformed into a linear system by output injection. Recently, [8] gave an algorithm which allows to recognize those nonlinear systems that can be steered to bilinear systems by output injection and nonlinear change of coordinates. In this paper, we are interested to a class of systems in the form:

(1) $$\begin{cases} dx/dt = [\Phi(y) + u.\Psi(y)].x + \Theta(u,y) \\ y = C.x \end{cases}$$

where x(t) represents the state variable, u(t) and y(t) are respectively the input and the output, Φ and Ψ are functions of the output y, Θ is a vector dependent on u and y and C is a constant matrix.

Notice that when Φ and Ψ are constant, we have a bilinear system with output injection studied in [8]. In our case, Φ and Ψ depend on the output and we call the system (1) an "extended" bilinear system with output injection.

An observer for (1) is a system which uses the input u(t) of (1) and the output y(t) of (1) and which leads a state estimation of (1). The general form of the observer equations is:

$$\begin{cases} dz/dt = f(z,u,y) \\ \hat{x} = g(z) \end{cases}$$

with f and g appropriate functions.

A natural observer for the system (1) is:

(2) $$dz/dt = \Phi(y).z + u.\Psi(y).z + K(t).(C.z - y) + \Theta(u,y)$$

K(t) is choosen so that: $\lim_{t \to +\infty} \| z(t) - x(t) \| = 0$.

The error estimation rate is:

(3) $$de/dt = (\Phi(y) + u.\Psi(y) + K.C) . e$$

with $y = C.x$ and $e = z - x$.

The system (3) is linear in relation to e(t) and e(t) = 0 is an equilibrium point of (3). The system (2) is an observer for (1) under condition that e(t) = 0 is stable. It is, therefore, sufficient to analyse the null solution stability.

3. Observers construction

Consider the following system which generally describes the behaviour of a fermentation process:

(4) $$\begin{cases} dX/dt = (\mu - D).X \\ dS/dt = - \mu.X/R + D.(S_{in} - S) \end{cases}$$

X(t), S(t), D(t), μ(t), S_{in} and R represent respectively the biomass, the substrate, the dilution rate, the specific growth rate, the input substrate and the yield.

Under the following assumptions:

1) Known input D(t) such that $0 < D_{min} \le D(t) \le D_{max}$.

2) S_{in} and R known parameters.

3) S(t) is measured i.e. the output equation of (4) is: y(t) = S(t).

The purpose is to construct one system which leads an estimate of X(t).

Among the specific growth rate analytical formulations which are the most used in practice and literature, we have retained the Monod law. It is certainly the famous and the most used law because it has been found in many cases to satisfactorily describe bacterial growth. Indeed, we assume that:

$$\mu(t) = \mu_m.S(t)/(K + S(t)) \qquad \text{with } \mu_m \text{ and K known constants.}$$

To construct a typical observer (2) for system (4), we have to reduce (4) to an "extended" bilinear system (with: $\Theta(u,y) = 0$ in this case). The following change of coordinates is then considered:

$$\begin{cases} x_1(t) = S_{in} - S(t) \\ x_2(t) = X(t) \end{cases}$$

The system (4) is written:

(5) $$dx/dt = \Phi(x_1).x + D.\Psi.x$$

The new output equation is: $y = C.x$ with $C= [1\ 0]$ and $x(t) = [x_1(t)\ x_2(t)]^T$.

$$\Phi(x_1) = \begin{bmatrix} 0 & \dfrac{\tilde{\mu}(x_1)}{R} \\ 0 & \tilde{\mu}(x_1) \end{bmatrix} \quad \text{and} \quad \Psi = -\mathrm{Id}_2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

with: $\tilde{\mu}(x_1) = \mu_m.(S_{in} - x_1)/(K + S_{in} - x_1)$

Considering the equation:

(6) $$dz/dt = \Phi(x_1).z + D.\Psi.z + K(t).(C.z - x_1) \quad \text{with} \quad K(t) = [k_1(t)\ k_2(t)]^T$$

we express the following.

Theorem: Under the assumptions:

A_1) $k_1 < D_{min}$

A_2) $k_2 = - \mu_m . S_{in} / (K + S_{in}) . R$

A_3) $\mu_m . S_{in} / (K + S_{in}) - D_{min} \leq 0$

the system (6) is an observer for the system (5) in the sense:

$$\lim_{t \to +\infty} \| x(t) - z(t) \| = 0$$

Proof: Let $e = z - x$ denotes the estimation error, we have:

(9) $$de/dt = R(t).e$$

with:

$$R(t) = \begin{bmatrix} k_1(t) - D(t) & \dfrac{\tilde{\mu}(x_1(t))}{R} \\ k_2(t) & \tilde{\mu}(x_1(t)) - D(t) \end{bmatrix}$$

System (6) is an observer for system (5) if the equilibrium point $e(t) = 0$ of system (9) is stable. We want to find $k_1(t)$ and $k_2(t)$ (independant of time if possible) such that stability is guaranteed. Stability of the null solution is given by the following lemma.

Lemma: If the conditions A_1), A_2) and A_3) are verified then $e(t) = 0$ is stable.

Proof: Let $f_1(e_1,e_2)$ and $f_2(e_1,e_2)$ the convergences rates of the estimation error:

$$f_1(e_1,e_2) = [k_1(t) - D(t)].e_1(t) + \tilde{\mu}(t).e_2(t)/R$$

and $$f_2(e_1,e_2) = k_2(t).e_1(t) + [\tilde{\mu}(t) - D(t)].e_2(t)$$

We show that there exist a Lyapunov function $V(e_1,e_2) = 1/2 \cdot (e_1^2 + e_2^2)$ such that:

$$\partial V/\partial e_1 . f_1(e_1,e_2) + \partial V/\partial e_2 . f_2(e_1,e_2) \leq 0 .$$

The above inequality is due to hypothesis 1) and to supplementary hypothesis issued from the physical and realistic constraints verified by the system (4). In particular, we have:

$0 \leq \mu(t) \leq \mu_m$ with $\mu(t) = \mu_m . S(t)/(K + S(t))$ and $0 \leq S(t) \leq S_{in}$

We deduce that: $0 \leq \tilde{\mu}(t) \leq \mu_m . S_{in}/(K + S_{in})$

Considering conditions A_1), A_2) and A_3), we then guarantee the observer convergence.

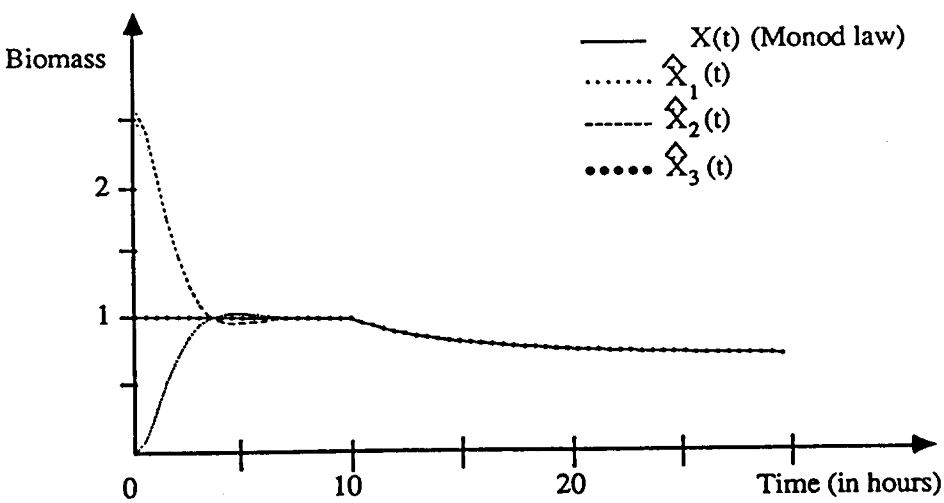
4. Numerical results

In this section, we use simulation experiments to illustrate the observers construction of bioprocesses models. The dilution rate D(t) is considered constant hence an universal input of the bioprocess basic model. The presented example is simulated with a Monod growth law and the following values.

Initial conditions of the model simulation: Maximum specific growth rate: $\mu_m = 0.9$, saturation constant: $K = 1.5$, initial substrate: $S(0) = 2$ (then $x_1(0) = 2$), initial biomass: $X(0) = 1$ (then $x_2(0) = 1$), constant dilution rate: $D = 0.51$, input substrate: $S_{in} = 4$, constant yield: $R = 0.5$, total duration of fermentation: $T = 30$

Initial conditions of the observer: We have choosen the parameter values such that they verify the conditions of the theorem. $z_1(0) = 2$, $k_1 = -1$, $k_2 = -1$ and $z_2(0)$ which takes 3 values: 0, 2.5 and 1.

In the figure below, we plot the biomass evolution (which is really not measurable) as a function of time and the estimates $\hat{X}_1(t)$, $\hat{X}_2(t)$, $\hat{X}_3(t)$ obtained by the observer. Any initial conditions for estimates have been taken. We see that the observer converges after only 4 hours and in spite of the disturbance (in order of 25 %) applied to the input substrate S_{in} at time = 10 h, it gives satisfactory results.



Estimation of X(t) from S(t) measurements.
(analytical specific growth rate)

5. Conclusion

This paper have dealt with nonlinear observers construction for microbial growth rate models in continuous fermentation processes. The proposed observer is asymptotic and unitialized. It provides estimates of the state variables which converge to the true values for all initialization of the observer. This is especially important for the bioprocesses where initial conditions are not measured. The main idea used here is to steer, by a change of state variables and output injection, the nonlinear basic bioprocess model to an "extended" bilinear system. We have then considered the case when the specific growth rate is given by an analytical expression (Monod law is taken as example) for which the state estimation problem has been solved. The observer convergence has been proved by existence of classical Liapunov functions. The effectiveness of the proposed observer is illustrated on numerical experiments. This observer present good performances with disturbing measures and can be generalized to other analytical specific growth rate formulations (like Contois law). One can prove [4] that when the specific growth rate is considered as a time varying parameter, it is only sufficient to know the $\mu(t)$ values obtained by on-line parameter estimation to guarantee the convergence of the observer.

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